course, the range of accuracy of the basic assumption of a slowly varying shear layer thickness in any of these cases remains to be determined. It is noted that if the shear layer thickness varies rapidly, i.e. $d\delta/dx$ is not sufficiently small, then the shear layer model itself becomes inapplicable.[4] Fortunately K becomes independent of δ^1 if $(x-x')/\delta(x) \gtrsim 1$ and δ itself does not vary significantly over a distance δ along the airfoil chord if $\delta/c \ll 1$.

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Three-Dimensional Compressible Stagnation Point Boundary Layers with Large Rates of Injection

G. Nath* and B. K. Meena†

Indian Institute of Science, Bangalore, India

Introduction

THE study of the steady laminar compressible boundary layers with large mass injection rates at a general three-dimensional stagnation point is of considerable importance in high-speed flight of spacecraft entering or re-entering planetary atmospheres. It is known that the large rates of injection in the boundary layer significantly alter the flow features from those usually expected. In particular, it is known that the boundary layer with large injection rates involves a structure consisting of a relatively thick inner layer close to the surface where viscous forces are negligible compared with pressure and inertia forces and of a relatively thin outer layer providing the adjustment of the inner layer to the inviscid external flow. The effects of moderate injection on the steady laminar compressible three-dimensional forward stagnation point boundary-layer flow of a gas with constant $\rho\mu$ flows ($\rho \propto T^{-1}$, $\mu \propto T$, $Pr=0.7$ where ρ, μ, T and Pr are the density, viscosity, temperature, and Prandtl number, respectively) have been studied by Libby[1] and with variable $\rho\mu$ flows ($\rho \propto T^{-1}$, $\mu \propto T^{\omega}$, $Pr=0.7$ where ω is the index of the power-law variation of viscosity) by Wortman and Mills[2] and Vimala and Nath.[3] Recently, for constant $\rho\mu$ flows, Libby[4] obtained an exact and an approximate solution of the above problem with large injection rates. The exact solution was obtained using quasilinearization techniques,[1] and the approximate solution using matched asymptotic expansions.[4] However, for variable gas properties (variable $\rho\mu$ flows), the solution with large injection rates has not been obtained before.

The aim of the present analysis is to obtain an exact solution of the preceding problem with variable gas properties and large injection rates. The governing equations were solved numerically using quasilinearization technique.[1] The results clearly indicate the inadequacy of solutions obtained under the simplifying assumption that $\rho\mu$=constant across the boundary layer especially for low-wall temperatures or large rates of injection.

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*Associate Professor, Department of Applied Mathematics. Present address: Department of Aeronautical and Astronautical Engineering, Ohio State University, Columbus, Ohio.

†Research Student, Department of Applied Mathematics.

Governing Equations

The governing equations in dimensionless form for the steady laminar compressible boundary-layer flow of a gas with variable properties in the neighborhood of a three-dimensional forward stagnation point with mass injection under similarity assumptions are[1-3]

$$f''' + (\omega-1)g^{-1}g'f'' + [(f+cF)f'' + g - f'^2]g^{1-\omega} = 0 \quad (1)$$

$$F''' + (\omega-1)g^{-1}g'F'' + [(f+cF)F'' + c(g-F'^2)]g^{1-\omega} = 0 \quad (2)$$

$$g'' + (\omega-1)g^{-1}g'^2 + Pr(f+cF)g'g^{1-\omega} = 0 \quad (3)$$

The boundary conditions are

$$f(0)=f_w,\ f'(0)=F(0)=F'(0)=0,\ g(0)=g_w \quad (4a)$$

$$f'(\infty)=F'(\infty)=g(\infty)=1 \quad (4b)$$

Here f and F are the dimensionless stream functions; g is the dimensionless enthalpy; g_w and $f_w = -(\rho w)_w/(\rho_e\mu_e a)^{1/2}$ are the dimensionless enthalpy and injection parameters at the wall, respectively (other symbols are given in Ref. 1). It may be noted that $\omega=0.5$ corresponds to the conditions encountered in hypersonic flight, $\omega=0.7$ corresponds to low-temperature flows, and $\omega=1$ represents the constant density-viscosity product simplification.[5] It is to be mentioned that most shapes of practical interest[1,4] range from sphere ($c=1$) to cylinder ($c=0$).

The skin-friction coefficients C_{f_x} and C_{f_y} along the x and y directions and the heat-transfer coefficient in terms of the Stanton number St are given by[1,3]

$$C_{f_x}=2(Re_x)^{-1/2}f_I''(0),\ C_{f_y}=2(R_{ex})^{-1/2}(v_e/u_e)F_I''(0) \quad (5)$$

$$St=(Re_x)^{-1/2}G'(0),\ f_I''(0)=g_w^{\omega-1}f''(0),$$
$$F_I''(0)=g_w^{\omega-1}F''(0) \quad (6)$$

$$G'(0)=Pr^{-1}g_w^{\omega-1}g'(0)/(1-g_w),\ Re_x=u_e x/\nu_e \quad (7)$$

where $f_I''(0)$ and $F_I''(0)$ are the skin-friction parameters along the x and y directions, respectively; $G'(0)$ and $F_I''(0)$ are the skin-friction parameters along the x and y directions, respectively; $G(0)$ is the heat-transfer parameter, and Re_x is the local Reynolds number. It may be noted that for $\omega=1$, $g_w^{\omega-1}=1(g_w>0)$ and $g_w=0.2$ and $\omega=0.5$ and $0.7, g_w^{\omega-1}=2.2361$ and 1.6207, respectively. It is clear that $g_w^{\omega-1}$ is an important parameter in predicting skin friction and heat transfer especially at low-wall temperatures. Therefore, the results obtained under the assumptions that $\rho\mu$=constant ($\omega=1$) across the boundary layer is of limited engineering value because, in effect, it eliminates the parameter $g_w^{\omega-1}$ from the skin friction and heat transfer.

Results and Discussion

The Eqs. (1-3) under conditions (4) have been solved numerically using a quasilinearization technique[1] for various

Table 1 Skin-friction and heat-transfer parameters for $c=0$ and $Pr=0.7$

		$g_w = 0.2$			$g_w = 0.6$		
$-f_w$	ω	$f_1''(0)$	$F_1''(0)$	$G'(0)$	$f_1''(0)$	$F_1''(0)$	$G'(0)$
0	1.0	0.7439	0.5139	0.6441	0.9976	0.5448	0.6791
0	0.7	0.8460	0.5676	0.7072	1.0533	0.5676	0.7056
0	0.5	0.9258	0.6078	0.7552	1.0902	0.5835	0.7241
0.5	1.0	0.4530	0.2249	0.3335	0.7258	0.2647	0.3822
0.5	0.7	0.5645	0.2809	0.4026	0.7854	0.2879	0.4105
0.5	0.5	0.6512	0.3235	0.4545	0.8274	0.3041	0.4303
1.0	1.0	0.2450	0.0543	0.1154	0.5189	0.0902	0.1700
1.0	0.7	0.3548	0.0962	0.1789	0.5784	0.1081	0.1958
1.0	0.5	0.4420	0.1313	0.2283	0.6205	0.1210	0.2140
1.5	1.0	0.1384	0.0041	0.0184	0.3792	0.0182	0.0537
1.5	0.7	0.2257	0.0181	0.0537	0.4331	0.0264	0.0707
1.5	0.5	0.3026	0.0357	0.0887	0.4719	0.0330	0.0835
2.0	1.0	0.0993	0.0001	0.0011	0.2922	0.0019	0.0111
2.0	0.7	0.1613	0.0015	0.0098	0.3377	0.0038	0.0182
2.0	0.5	0.2219	0.0058	0.0252	0.3713	0.0057	0.0242

Table 2 Skin-friction and heat-transfer parameters for $c=0.5$ and $Pr=0.7$

		$g_w = 0.2$			$g_w = 0.6$		
$-f_w$	ω	$f_1''(0)$	$F_1''(0)$	$G'(0)$	$f_1''(0)$	$F_1''(0)$	$G'(0)$
0	1.0	0.8209	0.7294	0.7690	1.0509	0.8683	0.8007
0	0.7	0.9268	0.8160	0.8426	1.1067	0.9106	0.8312
0	0.5	1.0092	0.8827	0.8982	1.1460	0.9403	0.8524
0.5	1.0	0.5194	0.4277	0.4438	0.7686	0.5819	0.4887
0.5	0.7	0.6339	0.5205	0.5236	0.8281	0.6267	0.5213
0.5	0.5	0.7226	0.5916	0.5836	0.8700	0.6582	0.5439
1.0	1.0	0.2888	0.2061	0.1902	0.5468	0.3663	0.2471
1.0	0.7	0.4039	0.2955	0.2689	0.6068	0.4100	0.2784
1.0	0.5	0.4940	0.3657	0.3286	0.6491	0.4409	0.3004
1.5	1.0	0.1526	0.0878	0.0444	0.3924	0.2297	0.0941
1.5	0.7	0.2496	0.1551	0.1006	0.4476	0.2669	0.1180
1.5	0.5	0.3319	0.2145	0.1497	0.4872	0.2939	0.1354
2.0	1.0	0.1008	0.0517	0.0042	0.2965	0.1581	0.0248
2.0	0.7	0.1679	0.0907	0.0239	0.3432	0.1861	0.0371
2.0	0.5	0.2334	0.1322	0.0514	0.3776	0.2072	0.0471
2.5	1.0	0.0797	0.0402	0.0002	0.2379	0.1219	0.0043
2.5	0.7	0.1295	0.0663	0.0036	0.2765	0.1430	0.0083
2.5	0.5	0.1795	0.0942	0.0129	0.3055	0.1592	0.0123
3.0	1.0	0.0665	0.0334	0.0000	0.1989	0.1006	0.0004
3.0	0.7	0.1076	0.0543	0.0004	0.2315	0.1176	0.0013
3.0	0.5	0.1483	0.0757	0.0024	0.2561	0.1305	0.0023

values of f_w, g_w, ω, and c taking $Pr=0.7$. The skin-friction and heat-transfer parameters $f_i''(0)$, and $G'(0)$ are given in Tables 1-3. The results for moderate or no injection rates have been included for the sake of completeness. It is evident from Tables 1-3 that these parameters increase as ω decreases but they decrease as f_w decreases. This is true for all values of g_w and c. It is also observed that these parameters are strongly dependent on ω (especially at low-wall temperatures) and f_w. It may be appropriate to mention that the parameters $f''(0)$, $F''(0)$, and $g'(0)$ which occur in the skin-friction and heat-transfer coefficients [Eqs. (5-7)] decrease as ω decreases, but $g_w^{\omega-1}$ increases as ω decreases. Consequently, as mentioned earlier, $f_i''(0)$, $F_i''(0)$, and $G'(0)$ increase as ω decreases. The parameters $f_i''(0)$, $F_i''(0)$, and $G'(0)$ also increase as c or g_w increases whatever may be the values of f_w and ω except that

Table 3 Skin-friction and heat-transfer parameters for $c=1.0$ and $Pr=0.7$

		$\omega = 1.0$		$\omega = 0.7$		$\omega = 0.5$	
$-f_w$	g_w	$f_1''(0) = F_1''(0)$	$G'(0)$	$f_1''(0) = F_1''(0)$	$G'(0)$	$f_1''(0) = F_1''(0)$	$G'(0)$
0	0.2	0.8957	0.8852	1.0069	0.9697	1.0931	1.0335
0	0.6	1.1102	0.9202	1.1670	0.9550	1.2070	0.9794
0.5	0.2	0.5893	0.5550	0.7084	0.6460	0.8002	0.7142
0.5	0.6	0.8217	0.6030	0.8819	0.6401	0.9242	0.6659
1.0	0.2	0.3449	0.2832	0.4655	0.3754	0.5592	0.4448
1.0	0.6	0.5885	0.3446	0.6494	0.3815	0.6923	0.4072
1.5	0.2	0.1824	0.0975	0.2900	0.1757	0.3781	0.2386
1.5	0.6	0.4183	0.1612	0.4754	0.1927	0.5161	0.2152
2.0	0.2	0.1072	0.0171	0.1853	0.0596	0.2585	0.1045
2.0	0.6	0.3082	0.0576	0.3573	0.0785	0.3932	0.0944
2.5	0.2	0.0805	0.0016	0.1341	0.0138	0.1895	0.0359
2.5	0.6	0.2419	0.0151	0.2822	0.0248	0.3123	0.0332
3.0	0.2	0.0667	0.0006	0.1086	0.0023	0.1514	0.0097
3.0	0.6	0.2003	0.0029	0.2336	0.0059	0.2588	0.0093

for $\omega \neq 1$, $G'(0)$ increases or decreases as g_w increases depending on the values of f_w and c. As expected, the velocity and enthalpy profiles (f', F', and g) at large injection rates clearly show the structure of the boundary layer; i.e., the boundary layer consists of a thin region with large gradients that separates the outer stream from a relatively thick inner layer adjacent to the wall where gradients are either very small or absent. For large injection rates, the velocity and enthalpy profiles have a point of inflexion for all values of ω, g_w, and c as is evident from a maximum in f'', F'', and g' (the profiles are not shown for the sake of brevity‡). Similar effects have also been observed by Gross and Dewey[5] and Vimala and Nath.[3] We have compared our results for $\omega=1$, $g_w=0.1$ and $-3 \leq f_w \leq 0$ with those of Libby[4] and for $\omega=1$, 0.7, 0.5; $g_w=0.2$, 0.6; and $f_w=0$, -0.5 with those of Wortman and Mills[2] and Vimala and Nath[3] and found them in excellent agreement.

Conclusions

The heat transfer and skin friction are strongly affected due to the variation of the density-viscosity product ($\omega \neq 1$) across the boundary layer especially for low-wall temperatures or large rates of injection which indicates that the linear viscosity-temperature relation ($\omega=1$) is not a good approximation for obtaining skin-friction and heat-transfer results in these cases. The skin friction and heat transfer are also strongly dependent on the injection rate and nature of the stagnation point.

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‡The profiles may be obtained from the authors.